

1 部分積分の計算

例 (1) $\int (x^2 + x + 1)e^{2x} dx$

$f(x)$	$g'(x)$			
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$x^2 + x + 1$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">e^{2x}</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\frac{1}{2}e^{2x}$</div> </div>	+	$(x^2 + x + 1) \cdot \frac{1}{2}e^{2x}$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$2x + 1$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\frac{1}{2}e^{2x}$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\frac{1}{4}e^{2x}$</div> </div>	-	$-(2x + 1) \cdot \frac{1}{4}e^{2x}$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">2</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\frac{1}{4}e^{2x}$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\frac{1}{8}e^{2x}$</div> </div>	+	$2 \cdot \frac{1}{8}e^{2x}$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">0</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\frac{1}{8}e^{2x}$</div> </div>			

$$(x^2 + x + 1) \cdot \frac{1}{2}e^{2x} - (2x + 1) \cdot \frac{1}{4}e^{2x} + 2 \cdot \frac{1}{8}e^{2x}$$

$$= \frac{1}{2}(x^2 + 1) \cdot e^{2x}$$

例 (2) $\int (\log x)^2 dx$

$f(x)$	$g'(x)$			
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$(\log x)^2$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">1</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">x</div> </div>	+	$x(\log x)^2$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">調整</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$\frac{2 \log x}{x}$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">1</div> </div>	-	$-2x \log x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$2 \log x$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">1</div> </div>	+	$2x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">調整</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">$\frac{2}{x}$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">1</div> </div>	+	$2x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">2</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">1</div> </div>	+	$2x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">0</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">x</div> </div>			

$$x(\log x)^2 - 2x \log x + 2x$$

例 (3) $\int e^x \sin x dx$

$f(x)$	$g'(x)$			
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">e^x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\sin x$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\cos x$</div> </div>	+	$-e^x \cos x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">微分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">e^x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\cos x$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\sin x$</div> </div>	-	$e^x \sin x$
<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">元に戻った</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: right;">e^x</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">×</div> <div style="text-align: left;">$\sin x$</div> </div>	<div style="display: flex; align-items: center;"> <div style="margin-right: 5px;">積分</div> <div style="border-left: 1px dashed black; height: 20px; margin-right: 5px;"></div> <div style="text-align: left;">$\cos x$</div> </div>	+	$-e^x \cos x$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x + \int e^x \cdot (-\sin x) dx$$

$$2 \int e^x \sin x dx = -e^x \cos x + e^x \sin x + C'$$

$$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$