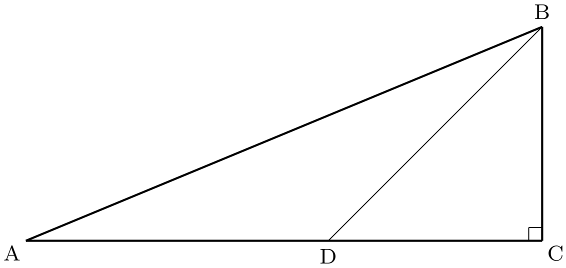
1　22.5°，15° の三角比を求める

1.1　22.5°

下の図の直角三角形 ABC で，$\mathrm{AD}=\mathrm{BD}=2$，$\mathrm{BC}=\mathrm{DC}=\sqrt{2}$ とする。



$\angle\mathrm{DAB}+\angle\mathrm{ABD}=\angle\mathrm{CDB}=45^\circ$ で $\mathrm{AD}=\mathrm{BD}$ であるから $\angle\mathrm{DAB}=22.5^\circ$

$\mathrm{AC}=2+\sqrt{2}$，$\mathrm{BC}=\sqrt{2}$ であるから，$\mathrm{AB}=2\sqrt{2+\sqrt{2}}$

$$\tan 22.5^\circ=\frac{\sqrt{2}}{2+\sqrt{2}}=\frac{\sqrt{2}(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})}=\frac{2\sqrt{2}-2}{2}=\sqrt{2}-1$$

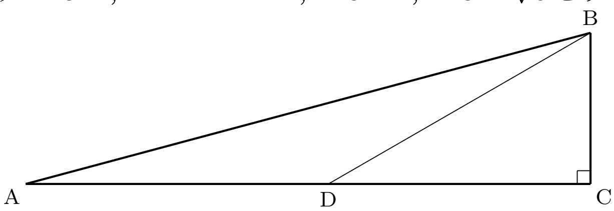
$$\cos 22.5^\circ=\frac{2+\sqrt{2}}{2\sqrt{2+\sqrt{2}}}=\frac{(\sqrt{2+\sqrt{2}})^2}{2\sqrt{2+\sqrt{2}}}=\frac{\sqrt{2+\sqrt{2}}}{2}$$

$$\sin 22.5^\circ=\frac{\sqrt{2}}{2\sqrt{2+\sqrt{2}}}=\frac{1}{\sqrt{2}\sqrt{2+\sqrt{2}}}=\frac{\sqrt{2}\sqrt{2+\sqrt{2}}}{2(2+\sqrt{2})}=\frac{\sqrt{2}\sqrt{2+\sqrt{2}}(2-\sqrt{2})}{2(2+\sqrt{2})(2-\sqrt{2})}$$

$$=\frac{\sqrt{2}(\sqrt{2+\sqrt{2}}\sqrt{2-\sqrt{2}})\sqrt{2-\sqrt{2}}}{4}=\frac{\sqrt{2}\sqrt{2}\sqrt{2-\sqrt{2}}}{4}=\frac{\sqrt{2-\sqrt{2}}}{2}$$

1.2　15°

下の図の直角三角形 ABC で，$\mathrm{AD}=\mathrm{BD}=2$，$\mathrm{BC}=1$，$\mathrm{DC}=\sqrt{3}$ とする。



$\angle\mathrm{DAB}+\angle\mathrm{ABD}=\angle\mathrm{CDB}=30^\circ$ で $\mathrm{AD}=\mathrm{BD}$ であるから $\angle\mathrm{DAB}=15^\circ$

$\mathrm{AC}=2+\sqrt{3}$，$\mathrm{BC}=1$ であるから，

$\mathrm{AB}=\sqrt{8+4\sqrt{3}}=\sqrt{8+2\sqrt{12}}=\sqrt{6}+\sqrt{2}$

$$\tan 15^\circ=\frac{1}{2+\sqrt{3}}=\frac{2-\sqrt{3}}{(2+\sqrt{3})(2-\sqrt{3})}=2-\sqrt{3}$$

$$\cos 15^\circ=\frac{2+\sqrt{3}}{\sqrt{6}+\sqrt{2}}=\frac{(2+\sqrt{3})(\sqrt{6}-\sqrt{2})}{4}=\frac{2\sqrt{6}-2\sqrt{2}+3\sqrt{2}-\sqrt{6}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin 15^\circ=\frac{1}{\sqrt{6}+\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}$$